

# Acorn Book Mult. Choice Assignment 17

1. I is false (sharp turn when  $x=c$  means it is not differentiable there)

II is true (no jumps, holes, or asymptotes)

III is true (positive slope means positive derivative -  $f$  is increasing)

## 2. Method 1

Recognize that this is the derivative  
of a cosine function with  $\frac{\pi}{2}$   
plugged in. (Limit Definition of the Deriv.)

$$\frac{d}{dx} \cos x = -\sin x$$

then plug in  $\frac{\pi}{2}$ ,  $-\sin \frac{\pi}{2} = -1$

## Method 2

L'Hopital's (treating  $h$  as a variable)

$$\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{2}+h) - \cos \frac{\pi}{2}}{h} = \lim_{h \rightarrow 0} \frac{-\sin(\frac{\pi}{2}+h)}{1} = -\sin \frac{\pi}{2} = -1$$

3. first and second derivatives negative means the function  
 is decreasing and concave down

4.  $y^3x + y^2x^2 = 6$  (Implicit differentiation - using prod. rule)

C  $y^3 \cdot 1 + x \cdot 3y^2 y' + y^2 \cdot 2x + x^2 \cdot 2y y' = 0$

$$3xy^2 y' + 2x^2 y y' = -y^3 - 2xy^2$$

$$y'(3xy^2 + 2x^2 y) = -y^3 - 2xy^2$$

$$y' = \frac{-y^3 - 2xy^2}{3xy^2 + 2x^2 y}$$

$$y'(2,1) = \frac{-1 - 4}{6 + 8} = \frac{-5}{14}$$

5.  $f(x) = x^4 - 2x^3$       CN  $x=0, \frac{3}{2}$       f'  $\begin{array}{c} - \\ \nearrow \\ 0 \\ \searrow \\ + \end{array}$       f''  $\begin{array}{c} - \\ \nearrow \\ 0 \\ \searrow \\ + \end{array}$       f''(x) =  $12x^2 - 12x$   
 C  $f'(x) = 4x^3 - 6x^2$       PPI  $x=0, 1$       f''(x) =  $12x(x-1)$   
 $0 = 2x^2(2x-3)$       PI at  $x=0, 1$

6.  $f(x) = \sin^2(3-x)$       This is chain rule.  $f'(0) = -2 \sin 3 \cos 3$   
 B  $f(x) = (\sin(3-x))^2$   
 $f'(x) = 2(\sin(3-x)) \cos(3-x)(-1)$

7. separate variables  $\frac{1}{2}y^3 = \frac{1}{3}x^4 + C_1$       when  $x=2, y=3$   $C_2 = 15$   
 E  $y^2 dy = x^3 dx$        $y^3 = \frac{3}{4}x^4 + C_2$        $3 = \sqrt[3]{\frac{3}{4} \cdot 16 + C_2}$   $y = \sqrt[3]{\frac{3}{4}x^4 + 15}$   
 $\int y^2 dy = \int x^3 dx$        $27 = 12 + C_2$

8.  $\int (x-1) \sqrt{x} dx$        $\frac{2}{5}x^{\frac{5}{2}} - \frac{3}{3}x^{\frac{3}{2}} + C$

D  $\int (x-1) x^{\frac{1}{2}} dx$   
 $\int (x^{\frac{3}{2}} - x^{\frac{1}{2}}) dx$

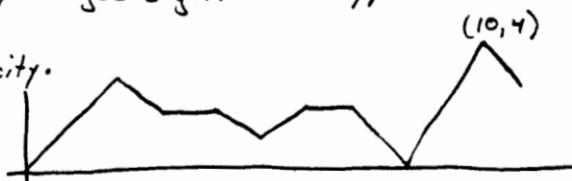
9. change direction means the velocity changes sign. This happens at  $t=8$ .

O

10. Speed is the absolute value of velocity.

E (It is never negative.)

A speed graph is shown.  
Greatest speed is at  $t=10$ .



11.  $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2} = \lim_{x \rightarrow \infty} \frac{x^2}{-4x^2} = -\frac{1}{4}$

B For end behavior, use the highest power terms

# Acorn Book Problems (cont.)

12.  $y = x \sqrt{4-x^2}$   
 B  $x=0, 2, \cancel{\infty}$



$$A = \int_0^2 x(4-x^2)^{\frac{1}{2}} dx$$

$$= -\frac{1}{2} \int_0^2 (-2x)(4-x^2)^{\frac{1}{2}} dx$$

$$= -\frac{1}{2} \cdot \frac{3}{3} (4-x^2)^{\frac{3}{2}} \Big|_0^2$$

$$\left( -\frac{1}{3} \cdot 0 \right) - \left( -\frac{1}{3} \cdot 4^{\frac{3}{2}} \right)$$

$$\frac{1}{3} \cdot \sqrt{4^3}$$

$$\frac{8}{3}$$

13. antiderivatives are integrals!

C  $\int \frac{\ln^2 x}{x} dx = \int (\ln x)^2 \cdot \frac{1}{x} dx = \frac{(\ln x)^3}{3} + C$  This constant  $C$  could be 0 or 6 (or any other number)

14.  $\frac{dy}{dx} = y \sec^2 x$

$dy = y \sec^2 x dx$

Separate variables

$\frac{1}{y} dy = \sec^2 x dx$

$$\int \frac{1}{y} dy = \int \sec^2 x dx$$

$$\ln|y| = \tan x + C_1$$

$$e^{\ln|y|} = e^{\tan x + C_1}$$

$$y = C_2 e^{\tan x}$$

now plug in  $y=5$  and  $x=0$  to find  $C_2$

$$5 = C_2 e^{\tan 0}$$

$$5 = C_2 e^0$$

$$5 = C_2$$

$$y = 5 e^{\tan x}$$

C

15.  $f_{\text{avg}} = \frac{\int_{-1}^1 e^{-x^2} dx}{2} = .747$  (using a calculator to integrate)

C

16. Washer Method  
 $V = \pi \int_0^1 ((2x)^2 - (2x^2)^2) dx = \pi \int_0^1 (4x^2 - 4x^4) dx$

B

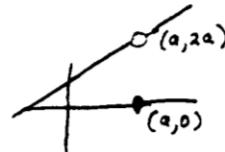
17.  $f(x) = \begin{cases} \frac{(x-a)(x+a)}{x-a}, & x \neq a \\ 0, & x=a \end{cases}$  The top piece has a hole when  $x=a \rightarrow (a, 2a)$ .  
 The bottom piece is a point  $\rightarrow (a, 0)$

D

I is true ( $\lim_{x \rightarrow a} f(x) = 2a$ , the value at the hole)

II is true  $f(a) = 0$ , the y value at  $(a, 0)$

III is false  $f$  is not continuous when  $x=a$  since the point does not fill in the hole



18.  $f''(x) < 0$  means  $f$  is concave down

D

Slope between first 2 pts =  $\frac{4.38 - 4.18}{1.2 - 1.1} = 2$

Slope between 2nd and 3rd pts =  $\frac{4.56 - 4.38}{1.3 - 1.2} = 1.8$

$f'(1.2) = \text{slope at } 1.2$

this would be between the two slopes above

19.  $x_1 = \sin t$

D  $v_1 = \cos t$

$x_2 = e^{-2t} - 1$

$v_2 = -2e^{-2t}$

same velocity means  $v_1 = v_2$

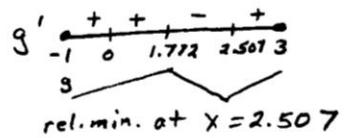
$\cos t = -2e^{-2t}$

Now graph with a calculator on the window  $[0, 10]$  and look for intersection points.

Acorn Book Problems (cont.)

20.  $g'(x) = \sin x^2$

$$\begin{aligned} \sin^2 x &= 0 \\ x &= 0, 1.772, 2.507 \end{aligned}$$



21. A, B, C and D all have symmetry so that an integral from  $-\pi$  to  $\pi$  would be zero (the negative signed area would cancel out the positive signed area)

$\cos x = x$  find pt. of intersection

$$x = .739$$

Washer Method  $V = \pi \int_0^{.739} ((\cos x)^2 - x^2) dx$  integrate with calculator

$$= 1.520$$

23.  $R(t)$  is a rate function in gallons per hour. To find the amount of oil in gallons we need to integrate. "After 10 hours" means from time 0 to time 10.

$$\int_0^{10} 2000 e^{-2t} dt = 8647 \text{ gallons}$$

24. I.  $\int_a^b f(x) dx$    II.  $\int_0^{b-a} f(x+a) dx$    III.  $\int_{a+c}^{b+c} f(x+c) dx$
- 
- Diagram I: A graph from  $a$  to  $b$  with a shaded area under the curve. An arrow points to it with the label "possible graph".
- Diagram II: A graph from  $0$  to  $b-a$  with a shaded area under the curve. An arrow points to it with the label "same graph shifted 'a' left a distance 'a'" and "same area".
- Diagram III: A graph from  $a+c$  to  $b+c$  with a shaded area under the curve. An arrow points to it with the label "shift left c" and "not the same area — the limits would have to be  $a-c$  and  $b-c$ ".

25.  $\frac{d}{dx} f(h(x)) = f'(h(x)) h'(x)$  but  $f'(x) = \frac{d}{dx} f(x) = g(x)$  and  $h(x) = x^2$  so  $h'(x) = 2x$

$$\begin{aligned} &= g(h(x)) h'(x) \\ &= g(x^2) 2x \end{aligned}$$

26.  $V(t) = \ln(t+1) - 2t + 1$

T.D. =  $\int_0^2 |\ln(t+1) - 2t + 1| dt = 1.540$