

Assignment 8 1997 MC (selected problems)

③ $\int_a^b (f(x) + 5) dx$

$$= \int_a^b f(x) dx + \int_a^b 5 dx$$

$$= a+2b + 5x|_a^b$$

$$= a+2b + 5b - 5a$$

$$= 7b - 4a \quad \boxed{C}$$

④ $f(x) = -x^3 + x + x^{-1}$

$$f'(x) = -3x^2 + 1 - x^{-2}$$

$$f'(-1) = -3(-1)^2 + 1 - (-1)^{-2}$$

$$= -3 + 1 - 1$$

$$= -3 \quad \boxed{D}$$

⑥ \boxed{C}

(velocity changes from positive to negative.)

⑦ tot. dist. = $\int_0^8 |v(t)| dt$

$$= \int_0^6 v(t) dt + \int_6^8 -v(t) dt$$

$$= 12 + 1$$

$$= 13 \quad \boxed{B}$$

⑧ f'

$$\begin{array}{c} - + + - \\ -2 \qquad 2 \end{array}$$

Shortcut:
 f' looks like a reg. quadratic so
 f must look like a neg. cubic

⑨ $f'(x) = \frac{14-x^2}{x-2}$

$$\begin{array}{c} - + - + \\ -2 \qquad 2 \end{array}$$

$$\text{dec. } (-\infty, 2) \quad \boxed{A}$$

⑩ pt. of tangency $(3, 2)$

slope of tangent is 5

tan. line eqn. is $y-2=5(x-3)$

Find the zero of the tan. line.

$$0-2 = 5(x-3)$$

$$-2 = 5x - 15$$

$$\frac{13}{5} = x$$

$$2.6 = x \quad \boxed{C}$$

⑪ Method 1

$$A = \int_{-2}^2 [5 - (x^2 + 1)] dx$$

$$= \int_{-2}^2 (4 - x^2) dx$$

$$(4x - \frac{x^3}{3})|_{-2}^2$$

$$(8 - \frac{8}{3}) - (-8 + \frac{8}{3})$$

$$16 - \frac{16}{3} \quad \boxed{D}$$

$$= 2(4x - \frac{x^3}{3})$$

$$= 2(8 - \frac{8}{3})$$

$$= 2 \cdot \frac{16}{3} \quad \boxed{D}$$

$$= 2 \cdot \frac{16}{3}$$

$$= \frac{32}{3} \quad \boxed{D}$$

⑫ $\int_0^{\frac{\pi}{2}} e^{\tan x} \sec x dx$

$$e^{\tan x}|_{0}^{\frac{\pi}{2}}$$

$$e^{\tan \frac{\pi}{2}} - e^{\tan 0}$$

$$e^1 - e^0$$

$$e - 1 \quad \boxed{C}$$

⑬ $f_{ave} = \int_{-3}^5 \cos x dx$

$$= \frac{\sin x|_{-3}^5}{8}$$

$$= \frac{\sin 5 - \sin(-3)}{8}$$

$$= \frac{\sin 5 + \sin 3}{8}$$

⑭ $\lim_{x \rightarrow 1} \frac{x}{\ln x} = \frac{1}{0} \quad \boxed{E}$

⑮ $f(x) = (x^2 - 3)e^{-x}$

$$f'(x) = (x^2 - 3)(-e^{-x}) + e^{-x}(2x)$$

$$= e^{-x}(-x^2 + 2x - 3)$$

$$= -e^{-x}(x-3)(x+1)$$

$$f' \quad \begin{array}{c} - + + - \\ -1 \qquad 3 \end{array}$$

inc. on $(-1, 3) \quad \boxed{D}$

⑯ $V = \int_1^e (\sqrt{\ln x})^2 dx$

$$= 1 \text{ integ. on calc.}$$

⑰ \boxed{C}

⑱ \boxed{C}

⑲ \boxed{C}

⑳ \boxed{C}

㉑ \boxed{C}

㉒ \boxed{C}

㉓ \boxed{C}

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㉟ \boxed{C}

1997 MC cont.

$$\begin{aligned} \textcircled{86} \quad f(x) &= \sqrt{x} \\ f(x) &= x^{\frac{1}{2}} \\ f'(x) &= \frac{1}{2} x^{-\frac{1}{2}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

rate of ch. at $x=c$ is twice rate of ch. at $x=1$

$$f'(c) = 2 \cdot f'(1)$$

$$\frac{1}{2\sqrt{c}} = 2 \cdot \frac{1}{2\sqrt{1}}$$

$$\frac{1}{2\sqrt{c}} = 1$$

$$\begin{aligned} \frac{1}{2} &= \sqrt{c} \\ \frac{1}{4} &= c \end{aligned}$$

A

$$\textcircled{87} \quad a(t) = t + \sin t \quad v(0) = -2$$

$$v(t) = \int (t + \sin t) dt$$

$$v(t) = \frac{t^2}{2} - \cos t + C$$

$$-2 = \frac{0^2}{2} - \cos 0 + C$$

$$-2 = -1 + C$$

$$-1 = C$$

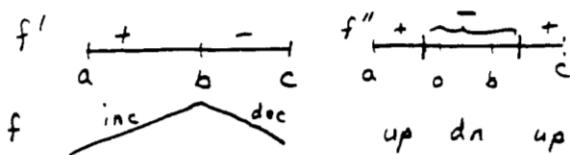
$$v(t) = \frac{t^2}{2} - \cos t - 1$$

$$0 = \frac{t^2}{2} - \cos t - 1$$

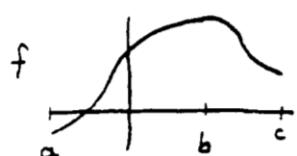
$$t = 1.478\dots \quad (\text{solving with a calculator})$$

B

$$\textcircled{88} \quad \text{Since } f(x) = \int_a^x h(t) dt, \quad f' = h$$



E



(A) is not possible since it has a sharp turn at $x=b$ so f' would not exist

$$\begin{aligned} \textcircled{89} \quad A &\approx \frac{1}{2} w (f(x_0) + 2f(x_1) + \dots + f(x_n)) \\ &= \frac{1}{2} \cdot \frac{1}{2} (3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) \\ &= 12 \end{aligned}$$

B